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Anomalous Thresholds and the Isgur-Wise Function

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Abstract

The original de Rafael-Taron bound on the slope of the Isgur-Wise function at zero recoil is known to be violated in QCD by singularities appearing in an unphysical region. To be consistent, quark models must have corresponding singularity structures. In an existing relativistic quark-loop model, the meson-quark-antiquark vertex is such that the required singularity is an anomalous threshold. We also discuss the implications of another anomalous threshold, whose location is determined by quark masses alone.

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The properties of QCD as the quark masses $m_b, m_c \to \infty$ [1] imply that the spectrum of the semileptonic decay $\overline{B} \to D^* \ell \overline{\nu}$ at the zero-recoil point $\omega \equiv v_B \cdot v_{D^*} = 1$ is absolutely normalized at leading order and receives no corrections at order $1/m_Q$ [2]. This gives rise to the possibility of a precise measurement of V_{cb} [3], provided *a)* the corrections beginning at order $1/m_Q^2$ are proven to be negligible, and *b)* a way can be found to extrapolate from the data at nonzero recoil $\omega > 1$ back to the zero-recoil point, where the rate vanishes kinematically. The latter requires knowledge of the shape of the Isgur-Wise function $\xi(\omega)$ which determines the spectrum at leading order and satisfies $\xi(1) = 1$. [In practice, it also requires knowledge of the magnitude and shape of the higher-order corrections, which are nonvanishing away from zero recoil even at order $1/m_Q$.]

In this letter we wish to look at the leading-order problem, namely the shape of the Isgur-Wise function. Some time ago, de Rafael and Taron derived a lower bound on its slope at zero recoil [4], the value of which is $\xi'(1) \geq -0.89$ [5]. This bound has since been shown to depend on assumptions which are not true in QCD. The bound also seems to be at odds with the data which favors a slope somewhat more negative than -1. We view any significant violation of the bound as providing an interesting clue to the underlying physics.

In particular it was pointed out by several groups [6] that the original derivation assumed that the b-number form factor $F(q^2)$ defined by

$$\langle \overline{B}(p') | \overline{b} \gamma_\mu b | \overline{B}(p) \rangle = F(q^2)(p + p')_\mu \tag{1}$$

is analytic in the region below the $B\overline{B}$ threshold at $q^2 = 4M_B^2$. The singularities corresponding to the three Υ's which lie just below threshold were ignored. These singularities can cause $\partial F/\partial q^2$ to become more positive at $q^2 = 0$. A quantitative estimate of the contribution of these singularities to the slope requires knowledge of the couplings of the Υ's to the vacuum and the $B\overline{B}$ pair, and involves large uncertainties [5, 6].

In the heavy-quark limit $m_b \to \infty$, the Isgur-Wise function is related to F by

$$\xi(\omega) = F(2M_B^2(1 - \omega)) \tag{2}$$

so the effect of the singularities is to allow $\xi'(\omega)$ to become more negative at $\omega = 1$. In the following, we will use ω as the variable. The $B\overline{B}$ threshold occurs at $\omega = -1$, and the original derivation of the bound assumed that $\xi(\omega)$ is analytic for all $\omega > -1$.

The authors of the first three papers of Ref. [6] also pointed out that it is possible to construct an example of a meson for which the slope would be large and negative even without the contributions of the Υ states. This is an artificial B meson for which the two quarks are unconfined and are bound in a Coulomb potential. In this case the mass of the B meson is greater than the sum of the masses of the two quarks. This produces a singularity due to an anomalous threshold in the region $-1 < \omega < 1$, which leads to a violation of the bound.

The present authors have recently developed a relativistic quark model of heavy-light mesons [7, 8]. When expanded in inverse powers of the heavy-quark masses the model is consistent with all constraints imposed by QCD via the heavy-quark effective theory, a property not shared by other popular quark models [9]. The Isgur-Wise function obtained in the heavy-quark limit of the model violates the de Rafael-Taron bound, and we wish to make clear how this can occur in a model of this type. This result may be contrasted with the simple quark loop model briefly described by de Rafael and Taron [4, 5], which happens to satisfy their bound.

The mechanism by which the bound is violated in our model is somewhat analogous to the artificial B meson mentioned above. The model contains a quantity which acts like a mass in the relevant three-point loop graph, and the sum of this mass and the heavy-quark mass is greater than the B-meson mass. The result is again a violation of the bound due to an anomalous threshold in the region $-1 < \omega < 1$.

The model represents the matrix element of Eq. (1) by a three-point quark loop graph with standard propagators for the quarks. The meson-quark-antiquark vertices contain damping factors proportional to

$$\frac{1}{-k^2 + \Lambda^2 - i\varepsilon}, \tag{3}$$

where $\Lambda << m_b$ and k is the momentum carried by the light quark. These factors act to suppress the flow of momenta larger than Λ into the light degrees of freedom, and the condition $\Lambda << m_b$ reflects the basic physical fact that the typical momenta in the light degrees of freedom are much smaller than the heavy quark mass.

The graph for the matrix element of Eq. (1) contains the product of a vertex factor and a light-quark propagator carrying the same momentum, which may be separated using partial fractions as

$$\frac{1}{-k^2 + \Lambda^2 - i\varepsilon}\frac{1}{-k^2 + m_q^2 - i\varepsilon} = \frac{1}{\Lambda^2 - m_q^2}\left(\frac{1}{-k^2 + m_q^2 - i\varepsilon} - \frac{1}{-k^2 + \Lambda^2 - i\varepsilon}\right). \tag{4}$$

The graph thus decomposes into a sum of graphs, in some of which the light-quark mass m_q is replaced by Λ. In the model the meson mass M_B is determined from the two-point function and it automatically satisfies the inequalities $m_Q + m_q < M_B < m_Q + \Lambda$. Thus Λ can play the role described above. As illustrative values we use $m_q \simeq 250$ MeV and $\Lambda \simeq 670$ MeV, where the latter is determined from a fit to the B, B^*, D, and D^* masses [7, 8].

Anomalous thresholds occur in the three-point graphs at values of ω for which *a)* all three particles internal to the graph are on their mass shell and *b)* the velocity of the heavy quark after interaction with the current is such that it can re-combine with the light degrees of freedom to form the final state meson. We have

$$\omega_{\rm anom} = 1 + \frac{[M_B^2 - (m_b + m)^2][M_B^2 - (m_b - m)^2]}{2M_B^2 m^2}, \tag{5}$$

where m can be either m_q or Λ and is independent of m_b. In the $m_b \to \infty$ limit, we may write $M_B = m_b + \overline{\Lambda}$, where $\overline{\Lambda}$ is the leading-order mass difference between the meson and the heavy quark. [It is calculable in the model in terms of Λ and m_q, and for the above values of the latter we find $\overline{\Lambda} \simeq 500$ MeV.]

The anomalous threshold corresponding to $m = \Lambda$ is located at

$$\omega_\Lambda = 2\frac{\overline{\Lambda}^2}{\Lambda^2} - 1 \tag{6}$$

In the model, $\overline{\Lambda}$ is always less than Λ, so this anomalous threshold occurs in the region $-1 < \omega < 1$. We could refer to this as a fake anomalous threshold since there is not a set of real particles going on shell.

Another anomalous threshold corresponds to $m = m_q$ and is located at

$$\omega_{m_q} = 2\frac{\overline{\Lambda}^2}{m_q^2} - 1. \tag{7}$$

$\overline{\Lambda}$ is always greater than m_q in the model, so the threshold occurs for $\omega > 1$. Its location is model-independent in the sense that it depends only on the heavy- and light-quark masses (since $\overline{\Lambda} = M_B - m_b$). It occurs in any model with unconfined quarks.

The Isgur-Wise function is completely determined in our model by the dimensionless ratio m_q/Λ, which can take values between 0 and 1. We find that the slope at zero recoil satisfies $\xi'(1) < -1.25$ for all values of m_q/Λ, thus violating the de Rafael-Taron bound. We show in Fig. 1 a plot of $-\xi'(1)$ versus m_q/Λ. For the illustrative values of m_q and Λ quoted above we find $\xi'(1) = -1.28$, and the slope is insensitive to variations in m_q/Λ over a considerable region around this point.

Fig. 2 shows $\xi(\omega)$ in the region $-1 \le \omega \le 1$ for our illustrative value of m_q/Λ. The singularity lying between $\omega = 0$ and 1 is the Λ anomalous threshold. The singularity at $\omega = -1$ corresponds to the $B\overline{B}$-threshold. In Fig. 3 we plot $\xi(\omega)$ for $\omega \ge 1$ to show the appearance and location of the m_q-threshold. It is interesting to contrast this step-function type singularity to the pole-like singularity occurring below $\omega = 1$.

When $m_q/\Lambda \to 0$ the m_q-threshold moves off to $\omega = \infty$, but $\overline{\Lambda}/\Lambda \to 0.72$ and the Λ-threshold remains near $\omega = 0$. Since the bound is still violated in this limit, this suggests that the Λ-threshold and not the m_q-threshold plays the dominant role in the violation of the bound. For $m_q/\Lambda \to 1$ we find $\overline{\Lambda}/\Lambda \to 1$. The two anomalous thresholds move together towards $\omega = 1$, and the slope of the Isgur-Wise function becomes very large and negative.

That the anomalous thresholds are intimately related to the slope of the Isgur-Wise function is also illustrated by an example described in [7]. There $\overline{\Lambda}$ was artificially set to zero holding everything else fixed, which corresponds to taking the heavy meson off-shell to the point $p_B^2 = m_b^2$. In this case the resulting slope was found to satisfy the de Rafael-Taron bound. In fact the whole Isgur-Wise function reduces to

a sum of two simple functions, one of which also appeared in the simple de Rafael-Taron quark model. In this artificial case, both anomalous thresholds coincide with the $B\overline{B}$-threshold at $\omega = -1$ [as seen in Eqs. (6) and (7)], and thus no longer lead to a violation of the bound.

The foregoing discussion has been independent of any particular semileptonic decay, since the Isgur-Wise function $\xi(\omega)$ is universal. But for a given decay, there is a finite physical region from $\omega = 1$ to $\omega_{\rm max} = (M_1^2 + M_2^2)/2M_1M_2$, where M_1 and M_2 are the initial and final meson masses. Depending on the value of m_q/Λ, the m_q-threshold could potentially be in this physical region. For the preferred values of m_q and Λ quoted above, however, $\omega_{m_q} \simeq 7$ which is far beyond the physically-accessible endpoint $\omega_{\rm max} \simeq 1.6$ for $\overline{B} \to D\ell\overline{\nu}$. We find that the m_q-threshold would occur in the physical region for this decay only if $m_q/\Lambda > 0.8$. The slope of the Isgur-Wise function would then be less than -2.3, in disagreement with the data. We see that physically reasonable results are obtained in the model as long as the anomalous thresholds do not lie too close to the physical region.

This observation is not surprising due to the fact that the model does not incorporate confinement (other than simply dropping imaginary parts of amplitudes). The model can be expected to give reasonable results in physical regions only if it is true that confinement plays little role in determining the values of such quantities. This would clearly not be the case if the anomalous thresholds, which reflect the presence of free quarks, showed up in physical regions.

We may consider physical processes away from the heavy quark limit and ask for what quark masses will the m_q-threshold lie outside the physical region. This leads to a more general three-point function with a meson with mass M_1 and heavy quark m_1 going to a meson with mass M_2 and heavy quark m_2, with the third particle in the loop having mass m. For a given pair of mesons the location of the m_q-threshold depends only on quark masses (with $m = m_q$) and it occurs at

$$\omega_{\rm anom} = \frac{1}{4M_1M_2m^2}\left\{(M_1^2 - m_1^2 + m^2)(M_2^2 - m_2^2 + m^2) + \Delta^{1/2}\right\} \tag{8}$$

where

$$\Delta = [M_1^2 - (m_1 + m)^2][M_1^2 - (m_1 - m)^2][M_2^2 - (m_2 + m)^2][M_2^2 - (m_2 - m)^2]. \tag{9}$$

[Eq. (5) is a special case of Eq. (8).] From this we find for the decay $\overline{B} \to D\ell\overline{\nu}$ the allowed region in the m_b-m_c-plane shown in Fig. 4, for $m_q = 250$ MeV. The allowed region includes most of the presently favored range for the quark masses. The allowed region is even larger for the decay $\overline{B} \to D^*\ell\overline{\nu}$.

In other decays it is possible that the quark masses are such that an anomalous threshold does lie in a physical region, in which case our model would offer a poor description. Confinement would at least smooth out the structure illustrated in Fig. 3, and possibly obliterate it without leaving a trace. The latter phenomenon apparently occurs in the solvable two-dimensional 't Hooft model. In this model the meson

masses may be calculated exactly in terms of the quark masses, thus determining the location of would-be anomalous thresholds. But in Fig. 6 of [10] no observable effect of such thresholds appears in the Isgur-Wise function.

In the real world we may consider decays having larger $\omega_{\rm max}$, such as $B \to \rho \ell \nu$ for which $\omega_{\rm max} \simeq 3.5$. The existence of an anomalous threshold in the physical region depends strongly on the effective light-quark mass. Inserting $m_1 = 4800$ MeV and $m = m_2 = 250$ MeV in Eq. (8) yields $\omega_{\rm anom} = 4.7$, while $m = m_2 = 330$ MeV gives $\omega_{\rm anom} = 2.3$. The latter is well inside the physical region. The existence of some remnant of such a threshold appearing in the data remains an intriguing, although slight, possibility.

In summary, the violation of the de Rafael-Taron bound constrains the singularity structure of models of heavy meson decay. We have shown how the required singularities can be produced in a model of unconfined quarks. The particular model we have studied gives reasonable results (i.e. confinement need not play an important role) as long as the singularities are well outside the physical regions. The singularity structure is related to the form of the meson-quark-antiquark vertex factor in Eq. (3). We therefore conclude that in models of this type the vertex factor is constrained by the requirement that the Isgur-Wise function be steep enough at zero recoil. Exponential vertex factors, for example, would fail to provide an anomalous threshold analogous to the Λ-threshold, and the Isgur-Wise function would be correspondingly shallow at $\omega = 1$.

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FIGURE CAPTIONS

FIG. 1: Negative slope of the Isgur-Wise function at zero recoil as a function of the ratio of the effective mass m_q of the light degrees of freedom to the scale Λ of typical loop momenta.

FIG. 2: Pole-like anomalous threshold in the region $-1 < \omega < 1$ which makes the slope of the Isgur-Wise function at $\omega = 1$ more negative than the de Rafael-Taron bound.

FIG. 3: Step-like anomalous threshold in the Isgur-Wise function above $\omega = 1$. [Note: the vertical scale is different from that of Fig. 2.]

FIG. 4: Region in (m_c, m_b)-space [below and to the left of the curve] for which the upper anomalous threshold lies above the maximum recoil point for $\overline{B} \to D\ell\overline{\nu}$.

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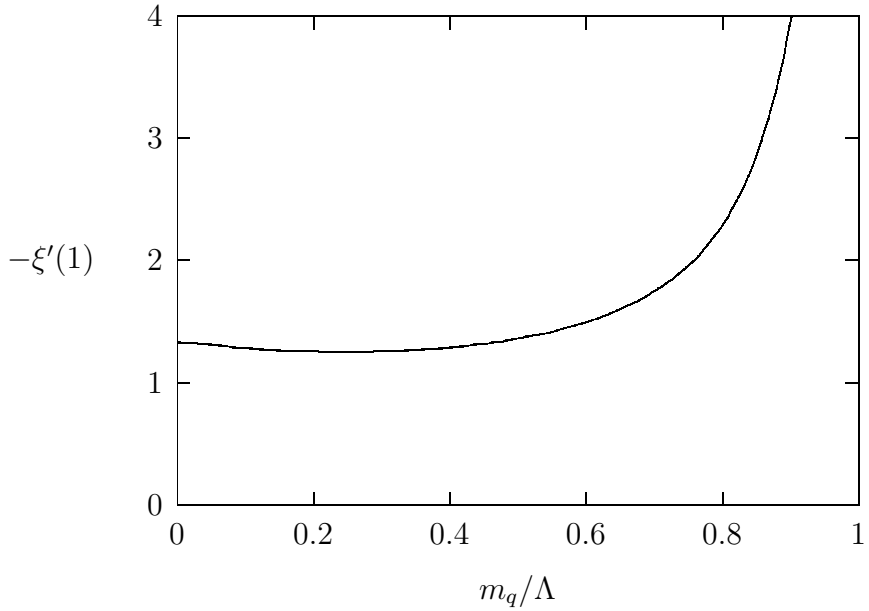

Figure 1

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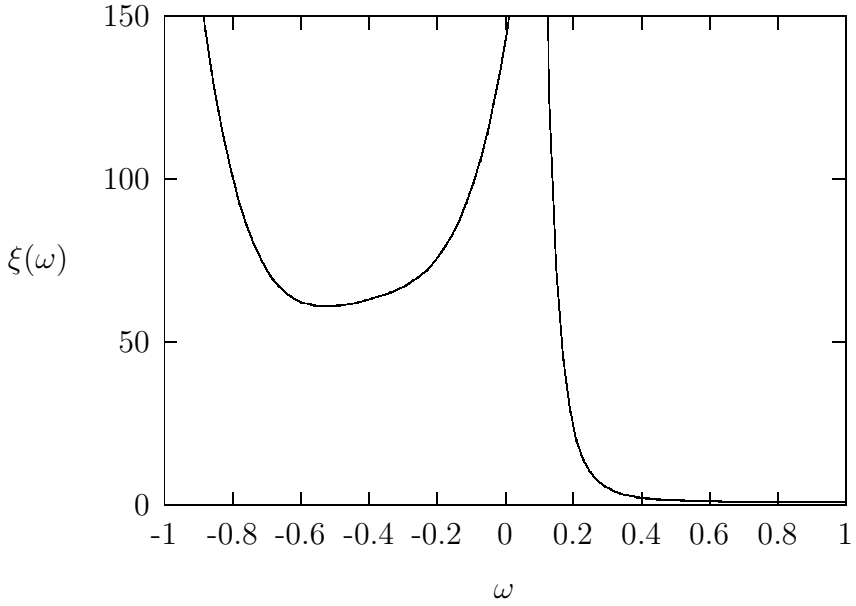

Figure 2

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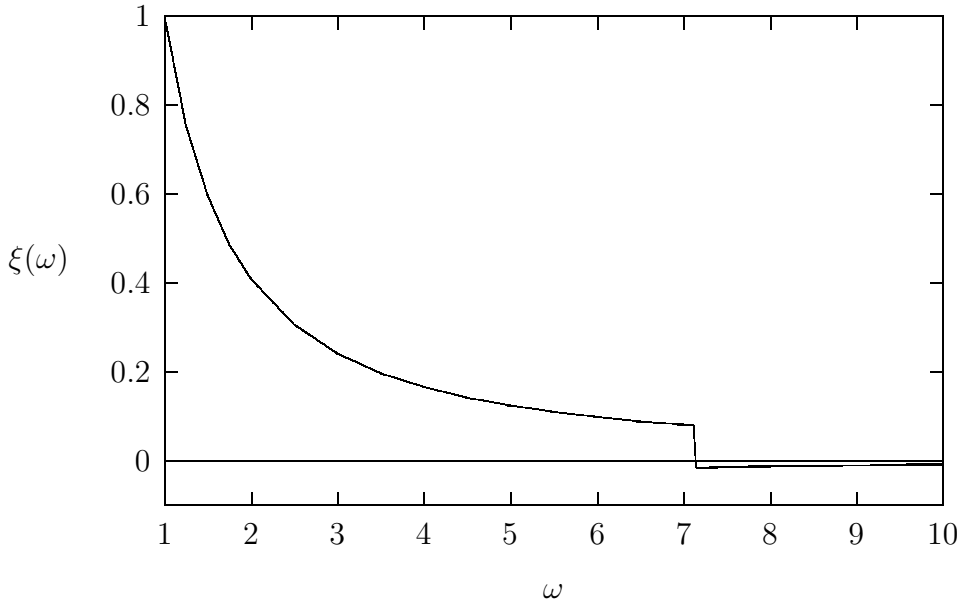

Figure 3

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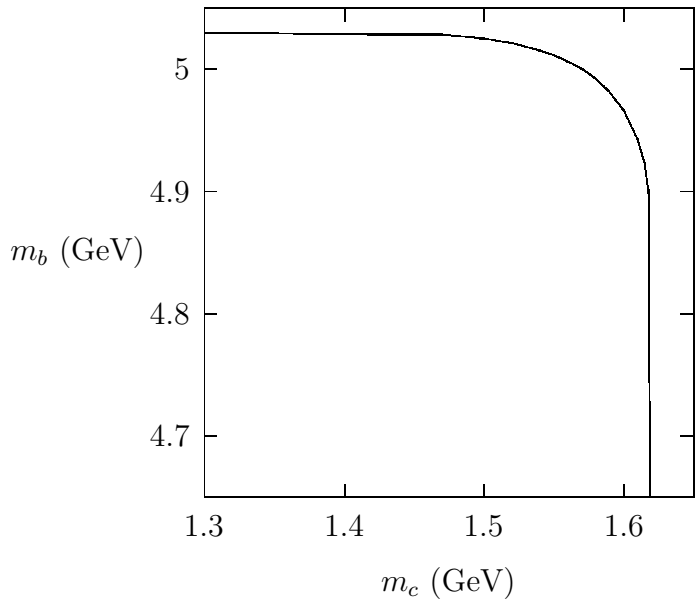

Figure 4